Science as Multiple Representations: Integrated perspectives on the role of learning and appropriating representations in constructing science understanding

A Qualifying Paper

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ABSTRACT

The domain of science is shaped by the development and use of representations of the concepts that explain the world in which we live. In other words, the language of science is that of representation. When children begin to make sense of science ideas, they do so through interactions with multiple forms of representation. Be it speech, written language, graphical notations, or gesture, the centrality of representation in science is undeniable. However, the conventional systems of representation that expert scientists use are systems that children must come to understand while making sense of the natural world. Thus, scientific understanding develops concurrently with knowledge of representation. This work explores how students learn and appropriate representations in the development of understanding. It is driven by four central questions that address: (1) the reasons for why representation is central to the development of thought; (2) a definition of representation in a manner such that it becomes a useful construct for studying science learning; (3) the evolution of conventional systems of representation and how these attempts to put the world on paper have impacted domains of knowledge; and (4) the literature on representation in science specifically from the standpoint of children spontaneously externalizing knowledge across multiple systems. The integration of theoretical constructs with empirical evidence supports the argument that students must be given opportunities to express themselves in different ways in order to understand science. Researchers must attend to the relationships between specific systems of representation and conceptual aspects when children engage in scientific discovery. This work calls for future research into how spontaneous and idiosyncratic representations of knowledge help students refine explanations and construct scientific understanding.
1.0 INTRODUCTION

The intent of this paper is to explore issues of representation in the development of scientific knowledge in the child. While representation is a widely used term in education, psychology, and child development research, the present work focuses primarily on the externalizations that children produce. The domain of science is one where external expressions of concepts, such as drawings, graphs, mathematical symbols, and written language, fuel the development of new ideas and innovation. Scientists rely on these modes of expression when attempting to demonstrate concepts through the development and refinement of models. They use various forms of externalizations to generate a model, thus, the ways in which scientists communicate ideas is intimately dependent on representation. For example, we have overwhelming empirical data suggesting that two massive bodies are attracted to each another by a gravitational force. However, before the apple fell on Sir Isaac Newton’s head, gravitational force existed. It was not until he developed a model for this concept that it could become part of the scientific discourse. Newton developed a model, expressed through different external representations, that described the underlying concept of gravity. The concept of gravitational force has been debated and refined over centuries, and the constant invention, critique, and revision of the models that explain gravity have lead to a deeper understanding of the concept. Arguably, it is the design and revision of models based on newly discovered evidence that fuels all new scientific discovery. Inherent in the revision of models is development of new ways to represent the ideas of science. The importance of modeling in science is widely recognized, and the way in which models are developed is through the use of external representations, which are the focus of this paper. When scientists are debating the intricacies of phenomena, they are
arguing about graphs, mathematical equations, drawings, diagrams, written language, spoken language, and even gesture. Thus, the language of science is representation. In other words, scientists “talk” through a multitude of representations, thus, an examination of the role representation plays in how children construct knowledge is crucial to the field of science education.

The goals of this paper are to explore the importance of research on representation in learning, to define representation in science education, to consider the evolution of representations throughout human history, and to review the literature on the learning and appropriation of external representations. A great body of literature on the topic of representation exists, which, in part, addresses science, mathematics, and language. It is imperative to include work from these various domains in considering how a child learns and uses externalizations of knowledge, because the child is immersed in a world inundated with a variety of representations. This work draws from multiple domains and theoretical perspectives and is guided by four essential questions:

1. *Why representations?* Why is representation an effective or interesting lens for studying the development of science concepts amongst learners of any age?

2. *What is representation?* How do the fields of education research, psychology, and child development define and use the concept of representation in order to inform theory and guide research?

3. *How do representations evolve and change historically in a social group?* How have representations historically impacted science and mathematics knowledge, and can we use historical accounts to inform our understanding of the development of the ability to represent in children?
4. How do we learn representations and how do we appropriate them? What has research in mathematics and science education taught us about how children learn representations and how they learn to appropriate them in educational contexts?

1.1 Vignette: Representing the Science of Parachutes

The essential questions driving this review are best contextualized for science education through an example of two after-school sessions with elementary school boys discussing parachutes and developing representations regarding “how they work.” After nearly thirty minutes of dropping toy parachutes with a cohort of third through fifth grade students (see Figure 1), the students had arrived upon two, well-articulated observations. First, they all agreed that

![Toy parachute](image)

Figure 1. Toy parachute used in after-school sessions with fifth grade students.

the parachute decreased the speed of descent for the attached load (e.g., the LEGO\(^1\) figure, a human, an air-lifted package, etc.). When dropping the LEGO figure without a parachute, it fell

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\(^1\) LEGO™ is a toy company that makes miniature figures that resemble human beings. The figures are approximately one inch tall, and they served as the payloads on the parachute replicas used in the described activities.
more rapidly to the ground than when the parachute was attached. The second observation was that the parachute did not fall straight down; it often fell to left or to the right, and sometimes it oscillated. Armed with these observations, the students were tasked with creating stop-action movies in order to help explain “how a parachute works.” James and Stephen, two fourth grade students, generated an animation that incorporated the following text, “Air gets trapped [sic] in the chute” (see Figure 2). This same group offered that the reason for the parachute’s angled descent was that perhaps one side of the LEGO figure weighed more, thus gravity would pull harder on that side of the parachute, causing it to fall in a particular direction.

![Figure 2. Screenshot from James and Stephen’s animation about parachutes.](image)

Alternatively, Jackson (also a fourth grader) described the workings of a parachute by first showing an individual paying money to a plane, then flying into the air, then jumping from the side door of the plane. He finished his animation by including the following text, “This is why parachutes work: #1 They save you from the fall; #2 If your parachute breaks you have
another one; #3 The gear save you from the altitude” (see Figure 3). For Jackson, the question of how a parachute works had quite a different meaning than it did for James and Stephen. Jackson seemed to understand the activity as one in which he was to describe how the act of skydiving transpires. Interestingly, in a conversation with Jackson after this session, he commented that he had gone skydiving with his father the previous summer. He shared what you would have to do if you wanted to test a parachute (i.e., pay for your ride, jump from the door), without attending to the science that might explain the fall. James and Stephen, who had never been skydiving, tried to address the ways in which science might explain how a parachute falls. Both animations are examples of students representing their understanding about an action they observed, within a particular system of representation: stop-action animations. However, clearly the prior experience of an activity as intense as skydiving made a monumental impact on how Jackson viewed a parachute. For James and Stephen, their interpretation of the task focused on how science could explain the parachute fall. Further investigation of their ideas through static
drawings, a different system of representation, unearthed additional understandings that James and Stephen held.

Figure 4. James’ parachute drawing, introducing the idea of an “x-ray” to “see” inside the parachute.

Figure 5. Stephen’s parachute drawing with accompanying written language description.

Both students, in a separate session, were asked to recall the parachute activity and to draw, individually, how they thought parachutes worked. James drew a picture of a parachute attached to a figure (similar to the LEGO figure), but also drew an “x-ray” window to display that the air “trapped” inside the parachute was turbulent (see Figure 4). He did this by drawing circling arrows inside the x-ray window. Stephen also drew a parachute, and he indicated that air moved past the person using arrows. Both students used a conventional representation for movement arrows but also introduced other means of expression such as written language. Stephen also relied heavily on a text description alongside the image to describe his reasoning behind the example (see Figure 5). In both cases, James through his x-ray representation and Stephen through his use of natural language, additional understandings were conveyed through
their drawings than was demonstrated in their jointly-produced animation. This episode highlights the importance of representation in science learning but also the importance of culturally relevant systems, conventions, and norms in how children express their understanding.

In the animated representations, at the simplest layer, all students displayed knowledge of a possible procedure for parachuting (see Figures 2 and 3). The animated medium lends itself to depictions of processes, even though the content of the movies are static images. However, with the drawings, a depiction of state is more prevalent than descriptions of process. That is, in drawing representations, the students tended to highlight aspects of the parachute-person system at a particular moment in time. Even though the animations provided them with the opportunity to express their understandings in each one of the static images in the sequence, it was in the drawings themselves that the students paused to elaborate on their ideas. In a sense, the children seem to be choosing to express different kinds of things in different systems, according to the underlying logic of the system: the drawing is static and closer to a state, the animation is a sequence and is closer to a process (see Figures 4 and 5). The animations showed the students’ understanding of procedure and process, including some scientific information, albeit through a combination of science and personal experience. But the drawing is where many of them displayed a deeper understanding of the inner workings of the parachute system. Thus, while the episode highlights the importance of representations in science, it also illuminates the importance of using different systems of representation to express or externalize understandings, a theme that will be elaborated on throughout this paper.
1.2 Initial Definitions of Representation

Jean Piaget’s *The Origins of Intelligence in Children* (1936/1977) contains a vignette of his 16-month old daughter Lucienne and the development of her ability to represent an idea. The episode exposes the mystery and power behind how representations impact thought and the construction of knowledge. Representation is a somewhat ubiquitous term in many fields of social science research, with a variety of definitions and usages. Kaput (1985) suggests that representation is an “undefined primitive whose meaning unfolds gradually through usage within a particular domain of inquiry” (p. 38). While in the English language the term “representation” is used in a number of different ways, making it difficult to define, other languages have developed different designations for *representation*, which clarify its meaning in a particular usage. The literature suggests a diversity of definitions for *representation*: Enyedy (2005) offers that representation is “the act of highlighting aspects of our experience and communicating them to others and ourselves” (p. 427); Goldin and Shteingold (2001) suggest that a representation is “typically a sign or a configuration of signs, characters, or objects...the important thing is that it can stand for (symbolize, depict, encode, or represent) something other than itself” (p. 3); and Lee and Karmiloff-Smith (1996) affirm the notion that representation “establishes a ‘stand for’ relationship between referent and sign” (p. 127; see also Kaput, 1991, 1998). Representations are often considered from two perspectives, internal and external (Goldin, 1998; Zhang & Norman, 1994). However, this distinction may be spurious (Nemirovsky, in press), thus we will first consider representation in a broader sense, as constituting some measure of “stand for” relationship.
1.3 The Origins of Representation in the Young Child

Under careful observation by Piaget (or perhaps by his wife, Valentine), young Lucienne is presented with a cardboard matchbox containing a watch chain. The lid to the matchbox is open as widely as it can be, and the chain, clearly visible, is coiled inside the box (where Lucienne has placed it in an earlier episode). She takes the box and immediately inverts it, allowing the chain to fall out. She has prior experience removing objects from receptacles; she has already practiced filling and emptying her pail. For Lucienne, the filling and emptying of a container is a scheme that she has developed through previous experiences. She uses a familiar action, one previously explored, to find success in a new situation. However, Lucienne is then presented with a box where the lid has been partially closed, leaving only a 10 mm opening. She repeats the action of turning the box over, but nothing spontaneously falls from the opening. She attempts to pull what little chain she can grasp, but still fails at removing the object from the box. Lucienne returns the box to its upright position and uses her index finger to retrieve the end of the chain and pull the remainder from the box. After this success, she is presented with the same box containing the chain, only the opening has been reduced to 3 mm, a gap smaller than her finger. Lucienne attempts to grasp the chain, but fails in her repeated trials. According to Piaget, at this point she “only possesses the two schemata: turning the box over in order to empty it of its contents, and sliding her finger into the slit to make the chain come out” (Piaget, 1936/1977, p. 238). She pauses, and during that pause we assume that she tries to think through the situation and to imitate the desired action that is necessary for success. Lucienne then produces the most delightful gesture: “she looks at the slit with great attention; then, several

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2 *Scheme* is used in the Piagetian sense, which is defined as follows: “A scheme is the structure or organization of actions as they are transferred or generalized by repetition in similar or analogous circumstances” (Piaget & Inhelder, 1966/1969, p. 4).
times in succession, she opens and shuts her mouth, at first slightly, then wider and wider!” (Piaget, 1936/1977, p. 238). Lucienne had already demonstrated an ability to imitate in the months leading up to this episode, and she uses this power of imitation to represent the future actions she wishes to achieve. The opening and closing of her mouth are a gestural representation of an action that she will perform with her hands only moments later, successfully retrieving the chain from the box. The kinesthetic representation was a gesture that aided Lucienne in solving the problem she confronted. Piaget uses this story to describe the genesis of representation in the child: representation being a momentous achievement in the life of the child, but also a pivotal development in the evolution of human beings.

Lucienne’s engagement with the matchbox brings to light the importance of representation in facilitating actions and thoughts. Before she had achieved the ability to represent the action of opening the box with her mouth, Lucienne had failed in attempts to retrieve the chain. As Nelson (in press) remarks, “representation is not the ‘natural’ first mode of thinking and does not occur in infancy” (p. 3). Prior to this episode, Lucienne lacked the ability to conceptualize a scheme for completing a task. However, the advent of representation provided her with this ability to think about the action she wished to achieve prior to physically performing it. Lucienne’s externalization of an idea helped her to develop a scheme for retrieving the chain from the box which she was incapable of doing prior to the aide of her representation of the required action. Opening and closing her mouth to represent an action elevated the complexity of cognition that Lucienne was able to achieve. This story underscores the importance of representation in assisting an individual’s attempts to develop methods and strategies for achieving goals.
1.4 The Importance of Representation

Lucienne’s development of the ability to represent was driven by a desire to know and understand the world. At birth, children possess a small set of reflexes that serve as the vehicle for interacting with physical objects (animate and inanimate) in the natural world (Piaget, 1936/1977). These reflexes allow for the young child to have new experiences and interact with new intriguing phenomena. In the days following birth, the child is already beginning to control these reflexes in exhaustive efforts to observe and understand the surrounding environment. Among other resources, such as those brought about by reciprocal assimilation, the young child uses external interactions with the outside world (such as grasping, touching, pushing, dropping) to construct knowledge. And, as shown with Lucienne, children soon develop the ability to represent their own and others’ actions adding a new dimension to such interactions with the natural world.

The development of language is a hallmark of the semiotic function, which occurs near the end of the sensorimotor stage of development (Piaget, 1936/1977), at around eighteen months of age. The semiotic function encompasses two forms of representation, symbols and signs. Piaget and Inhelder (1966/1969) offer that symbols “are motivated – that is, although they are differentiated signifiers, they do present some resemblance to the things signified,” and that signs are “arbitrary or conventional” (p. 57). Symbols are arbitrary in the sense that they have no natural connection with the signified (de Saussure, 1959), but rather are linked to the signified through some cultural convention. Operating in this new world of the semiotic function involves distinct differences between signs and symbols that have to do with conventionality.

3 I acknowledge the controversy surrounding this statement, especially as portrayed or enacted in the discussions between constructivists and nativists, first famously documented in the Piaget Chomsky debate (Piattelli-Palmarini, 1980) but continuing to this date with, for example, Spelke’s challenge (see Lipton & Spelke, 2006) of Piaget’s hypothesis regarding the genesis of number (Piaget & Szeminska, 1941/1977).
Conventionality requires an additional layer of involvement on the part of the child, as it requires the child to accept already created rules, as opposed to making up their own, as could happen in play (but not rule games) or dreams.

Representation is paramount in Piaget’s theory because the semiotic function is a major component of logical thought⁴ (i.e., the pinnacle of the development of the child). According to Gruber and Voneche (1977), “logical operations [mechanisms of thought] are not read directly off the environment: they are the product of reflection and abstraction by the subject upon his own actions and their coordination into a meaningful system” (p. 481). Essentially, representation enables the child to “locate himself in a space that goes beyond the perceptual space and in a field of time that permits the coordination of past, present, and future” (Gruber & Voneche, 1977, p. 485). This relocation paves the path for logical thought as the child begins the “decentering [sic] process whereby the child eventually comes to regard himself as an object among others in a universe that is made up of permanent objects and in which there is at work a causality that is both localized in space and objectified in things” (Piaget & Inhelder, 1966/1969, p. 13). Logical thought is dependent on an ability to think abstractly and to be reflective. The ability to represent one’s own ideas or those of another individual is, thus, essential in the process of developing logical thought. Piaget argues that representation is of importance to the child because it allows for the child to signify actions and ideas to him- or herself. In addition, representations have become, over time, part and parcel of different areas of knowledge, such that in order to become a relatively expert member of a particular content domain implies having

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⁴ It is not my intention to suggest that representation is only relevant to logical thought. Rather, I am using the term logical thought specifically as Piaget introduced it. Piaget offered that the semiotic function (i.e., capacity for representation) allowed for the construction of logical structures of mind such as seriation, classification, and conservation (Gruber & Voneche, 1977). Logical thought allows the child to see objects position in space as unimportant or temporal sequences as reversible (Gruber & Voneche, 1977). Thus, usage of the term logical thought in this paper serves to honor the theoretical positions of Piaget as they pertain to representation.
appropriated the associated representations. For instance, “knowing” algebra implies having appropriated the representations typically associated to this area of knowledge: algebraic notation, function tables, and Cartesian coordinate graphs. Furthermore, I will argue that representations are also important because they exist in a cultural context and in social situations; representations are a means of expressing ideas and externalizing understanding. Defining the importance of and arguing for the centrality of externalizing ideas in the learning of science is the driving force behind this paper.

1.5 Representation is Uniquely Human

The ability to represent ideas is, as Donald (1991) argues, a uniquely human capacity. From a historical perspective, Homo sapiens separated themselves from their closest ancestors based on their capacity for representation (Tomasello, 1999). With the capability to re-present experiences, thoughts, and ideas, the construction of knowledge based on interactions with the physical world takes on a new level of complexity and involvement in the development of intelligence. Representation allows for reflective thought (Tolchinsky-Landsmann & Karmiloff-Smith, 1992), for abstract thought which, in turn, allows humans to see others as intentional agents (Tomasello, 1999), for the comparison of ideas across contexts and situations (Behr, Lesh, Post, & Silver, 1983; Goldin & Shteingold, 2001), and for descriptive expressions of understandings that may otherwise remain not articulated (Martí & Pozo, 2000; Nelson, in press; Olson, 1994). Using this evolutionary perspective (that the capacity for representation sets

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5 The term understanding is used in reference to whatever the individual holds to be true. At all points of life, we strive to find understanding, and we are remarkably good at doing so. However, these understandings may not always be the same or similar (i.e., an understanding can “disagree”) with commonly held scientific concepts. Thus, I use the term understanding instead of concepts to distinguish between the idiosyncratic ideas children hold and the generally accepted concepts that scientists have come to accept.
humans apart from other species) to think about the development of the modern-day child helps us to realize the important role that representation plays in thought as well as in thought construction. If representation is what sets humans apart from other organisms, then it is imperative that we attend to how the child constructs, invents, and appropriates representations.

1.6 Representation: Process and Conceptual Object

The sense in which the term representation is used herein namely refers to externalizations (i.e., events or objects perceivable by others). External representations have an important duality that make them a valuable lens through which to analyze how children develop understanding. The distinction is based on an important duality captured in different ways by different theoretical frameworks (see Breidenbach, Dubinsky, Hawks & Nichols, 1992; Douday, 1997; Ferreiro, 1994; Sfard, 1991; Tolchinsky-Landsmann & Karmiloff-Smith, 1992) between process (or tool, in Douday’s terms) and object. From one perspective, the operations or actions required to represent a scientific or mathematical concept can be considered as a process. For example, in mathematics, a function can be seen as a “method for getting from one system to another” (Skemp, 1971, p. 246). This definition of function reflects an operational conception (Sfard, 1991) of the idea; functions are considered as a process through which to achieve some transformation. For children developing an understanding of functions, the approach to the concept is from a process perspective, where the function is a tool (Douday, 1997) for doing something. Similary, consider the development of written language. A child’s understanding of written language is a constructive process, where the use of idiosyncratic inscriptions and conventional letters are tools for conveying ideas (Brizuela, 2001). It is not until the child has
developed sufficient understanding of the written language system that they can begin to consider words as objects that represent ideas which can be reflected upon. Sfard (1991) argues that (in mathematics) “the majority of ideas originated in processes rather than objects” (p. 11). For children, the development of knowledge and of a capacity for representing understanding in various systems begins as a process as well.

Children grow up in a world that is filled with conventional and idiosyncratic systems of representation. They experience oral language, written language, graphs, gestures, pictures, moving-pictures, and a myriad forms of representations. Once they have developed the semiotic function\(^6\), they begin to experiment with and refine how they represent their understandings. Living immersed in a world full of symbols and signs, it is to be expected that this process of representing will be influenced by exposure to and interactions with various forms of representation—representations laying in a world “external” to the child and capturing a historical trajectory of some kind on the part of humankind. Furthermore, the process of representing as essentially referential-communicative (Tolchinsky-Landsmann & Karmiloff-Smith, 1992) yields a physical artifact or action perceivable by others (e.g., gestures, writing on paper).

These artifacts become conceptual objects in their own right (Olson, 1994), whereby the object, while being a representation, is percieved as the referential concept. This structional conception (Sfard, 1991) of representations allows the individual to reflect upon the concept as embodied in the external representation. Therefore, external representations can be both a process and conceptual objects. Representations as conceptual objects are pivotal in the development of thought, as placing an idea in the external world allows for reflection and evaluation of one’s understanding (Kaput, 1991). By making knowledge explicit through a

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\(^6\) At around 18 months of age, according to Piaget and Inhelder (1966/1969).
process of externalizing ideas, children strengthen and develop their conceptual understanding (Lehrer & Schauble, 2002; Schliemann, Carraher, & Brizuela, 2007; Tytler & Prain, 2007). In addition, the artifact (i.e., conceptual object) provides the researcher with a perspective on what the child knows (Kaput, 1998). Eventually, the external representation becomes a fundamental element of any understanding. Thus, representation is a process whereby children explicate their knowledge, resulting in a conceptual object that is part and parcel of thought.

2.0 Why study representation?

This section outlines arguments for why representation is a valuable lens through which to analyze how children come to learn ideas about science and to express their understandings. Furthermore, I will also argue that representing their ideas helps children (as well as adults) to elaborate further on their previously held understandings. I specifically consider how representations aide thought, how cultures develop conventional forms of representations that influence how children learn to externalize, how representations serve as a tool for negotiating shared meaning, and how children engage in both the process of representing and the use of representations as conceptual artifacts for investigating science.

2.1 Representations Help Us Think

The story of Lucienne outlined earlier highlights one of the essential motivations for the study of representation in education. For Lucienne, opening and closing her mouth represented the action she ultimately sought to replicate with her hands acting on the matchbox. From a more technical standpoint, the act of retrieving the chain from the box (i.e., opening the lid) was
the *signified*, and Lucienne’s kinesthetic gesture constituted the *signifier* (de Saussure, 1959; Piaget & Inhelder, 1966/1969). By developing a “stand-for” relationship (a concept discussed in greater detail below in Section 3.1), Lucienne demonstrated two Piagetian constructs: the semiotic function and the Copernican revolution. Lucienne’s symbolic gesture was an indication that she had begun to develop a *semiotic function*, or an ability to generate a signifier for representation’s sake (Piaget & Inhelder, 1966/1969). Early signs of the semiotic function also indicate Lucienne’s presence within a “Copernican revolution” (Piaget & Inhelder, 1966/1969), wherein she can recognize that she was but one object within a larger universe of many independent objects. Her perspective, at the point of the matchbox incident, has shifted from egocentrism toward one where she considers herself as an independent object interacting with another independent object (i.e., the matchbox) in a localized context. In other words, Lucienne has begun to understand that there is a universe beyond that which she can directly perceive. This process is also referred to as decentering\(^7\), and it affords Lucienne the ability to represent. Decentering allows Lucienne to recognize the existence of a causality that is “both localized in space and objectified in things” (Piaget & Inhelder, 1966/1969, p. 13), which forms the foundation from which she will develop reflective and, eventually, abstract thought. Thus, the act of representing (i.e., the process) helps us to organize and refine our ideas (Kaput, 1991), and the artifact (i.e., the object) becomes a vehicle for thought. Moreover, the representations we produce are encapsulated within our understanding of the natural world; they become part of our *minds*\(^8\).

\(^{7}\) Which Piaget would argue is a long, arduous process that may begin in the semiotic stage, but continues to occur well into the pre-operational stage (Piaget & Inhelder, 1966/1969) as well as beyond, including adulthood.

\(^{8}\) I use the term *mind* cautiously as it is the topic of great debate. However, it seems the most appropriate of terms to use in reference to the ideas we have and how we go about expressing them. To separate internal and external worlds gives a false impression that they are, in fact, separable (see Nemirovsky, in press). In this paper, I use the term *mind* in reference to internal cognition, acknowledging it is intrinsically linked with externalizations. I propose that understandings and representations are inseparable, and thus refer to the entirety of human thought as the *mind*. 
The perspective that representations are essential to formal thought is primarily a Piagetian construct (Piaget & Inhelder, 1966/1969), however, the idea has much broader implications. In the world of mathematics and science, the use of multiple forms of representation (both internal and external – see section 3.2) is heavily relied upon for the general activities of professionals in the field. Just as Lucienne used a representation of an action to assist her in the solving of a task, humans at every age use representations to aide and assist thought. More broadly, external representations have also been found to influence entire domains of knowledge, such as with the introduction of the Feynman Diagram to the field of theoretical physics. Richard Feynman’s proposal of this new representation shaped the thinking of quantum dynamics researchers, and the tale of how this new form became accepted as a conventional means for communicating quantum concepts is a vignette which illustrates many of the aspects of representation under consideration herein.

2.2 Representations in Physics: The Feynman Diagram

At the turn of the 20th century, research in physics focused on developing a theory to explain what matter is and how it might interact with light and energy. By 1926, an “uncommon-sensy” (Feynman, 1985, p. 3) theory was developed that was called quantum mechanics. The merging of this new theory for matter with ideas about how matter and light interact resulted in the development of quantum electrodynamics, or QED. The conceptual underpinnings of the theory were adopted by the physics community, but calculations within QED remained very confusing and complex leading up to 1948. At this time, three individuals simultaneously solved the puzzle of QED. The trouble with previous theoretical models for
QED lay in the accuracy of the calculations these models produced. Calculations of rough estimations yielded fairly accurate results when tested against experimental values; however, when more accurate computation was desired, the models produced quite large errors (Feynman, 1985). Julian Schwinger, Sin-Itiro Tomonaga, and Richard P. Feynman are credited with devising methods for deriving equations and performing calculations based on QED that, when compared with experimental results\(^9\), were quite accurate (Kaiser, 2005). Discussions of these new approaches to QED were the focal point of a series of post-World War II conferences that were designed to re-energize the theoretical physics community\(^10\) (Schweber, 1986). It was at the second of these conferences, held in the Poconos, where Richard Feynman first introduced a new representation to assist calculations in QED that would eventually “revolutionize nearly every aspect of theoretical physics” (Kaiser, 2005, p. 4), the Feynman Diagram.

Before discussing the unveiling of the Feynman Diagram at the Poconos conference, it is important to realize the robustness of QED and its place in modern theoretical physics\(^11\). The accuracy of QED is demonstrated by Paul Dirac’s work, where he employed the theory of relativity to calculate the strength of the magnetic moment of an electron\(^12\). Dirac’s number, as it became known, was originally calculated to be exactly 1 in particular units (Feynman, 1985). This number was purely theoretically based, and later experiment results showed that Dirac’s calculation needed a correction and that it was closer to 1.00118. Schwinger introduced new

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\(^9\) A discovery for which all three shared the Nobel Prize in physics in 1965.

\(^10\) Remember, the end of WWII saw the first, and only, use of an atomic weapon that was designed and built by many of the top physicists in the United States. Perhaps this effort to re-energize the physics community stemmed from attempts to raise many demoralized spirits after Nagasaki and Hiroshima.

\(^11\) I will not describe QED in sufficient detail for the reader to understand; rather, I will describe the role of representation in shaping physics research. As Feynman (1985) states, “It is my task to convince you not to turn away because you don’t understand it. You see, my physics students don’t understand it either. That is because I don’t understand it. Nobody does” (p. 9). This illustrates the complexity of these ideas and the precise reason why a complete description of QED is outside of the scope of this paper. The reader is encouraged, in this paper, to be mindful of the story of QED, not necessarily the theoretical implications themselves.

\(^12\) This is one of many examples used to show the accuracy of QED; it is not the only case where such precision occurred.
theoretical methods that calculated the same number to be 1.00116, which showed that the new QED work was on the right track, based on Dirac’s efforts. Feynman (1985) describes the process of validating QED like so:

Experiments have Dirac’s number at 1.00115965221 (with an uncertainty of about 4 in the last digit); the theory puts it at 1.00115965246 (with an uncertainty of about five times as much). To give you a feeling for the accuracy of these numbers, it comes out something like this: If you were to measure the distance from Los Angeles to New York to this accuracy, it would be exact to the thickness of a hair. That’s how delicately quantum electrodynamics has, in the past fifty years, been checked – both theoretically and experimentally. (p. 7)

To quote Feynman (1985) again, “There is no significant difference between experiment and theory” (p. 7), and this [lack of difference] demonstrates the magnitude of the impact QED had (and continues to have) on modern physics. QED has been compared with Einstein’s relativity as one of the most successful theoretical advancements in physics history (Gribbin, 2002). While time has gathered significant evidence in support of its accuracy against experiment, the ease with which physicists could use QED was severely limited until the introduction of Feynman’s Diagram at the National Academy of Science’s (NAS) Poconos conference in 1948.

The spirit of the Poconos conference (like those held at Shelter Island and Oldstone) was small, closed, and elitist (Schweber, 1986). Originally Duncan MacInnes, a member of the NAS, proposed the conferences as a means to “get out these younger guys” (Schweber, 1986, p. 302). Physics legends like Neils Bohr, Paul Dirac, and Robert Oppenheimer were present at these events; however, the real intent of these conferences was a venue for the younger physicists to make their mark, men like Schwinger, Feynman, and Freeman Dyson. The younger generation
brought new calculation techniques to quantum theories that increased the accuracy of these theoretical models and provided clues for some previously unanswered questions. The Poconos conference featured the virtuoso, and Feynman rival, Julian Schwinger presenting a nearly day-long lecture on his newly devised QED calculation methods which had already garnered acclaims at previous meetings (Kaiser, 2005). Feynman was to present his work following Schwinger’s, which was very well received. Given that Feynman’s approach was alternative to the mainstream, the odds were stacked against him because he followed a well-received presentation with much less time in which to present his radically different graphical technique. The calculations of QED were tedious and complicated and, as a result, frustrated many accomplished physicists. Motivated by these frustrations, Feynman developed special diagrams as a bookkeeping method for these complicated calculations. Realizing the power of these diagrams, Feynman chose to share them with the QED community at the Poconos conference.

Despite the warm reception that Schwinger received, the audience in the Poconos was fully aware that even Schwinger’s techniques failed to alleviate the confusion of using these standard, mathematical methods. While the group was eager for an improved method, they failed to see Feynman’s approach as the answer. His presentation was laced with doubt, interruptions, and questions and, as a result, the presentation lacked a clear, systematic, and justified introduction to the Feynman method (Kaiser, 2005). The diagram (see Figure 6) was unlike anything anyone had ever seen, and even Feynman admitted that they were rather unusual. The audience had strong objections to its appropriateness and validity as a scientific tool. The diagram helped to describe electron scatter, partially by representing positrons as electrons going backward in time (see Figure 6 legend). Neils Bohr vocalized a strong objection to this idea, arguing that the Heisenberg uncertainty principle prohibited the assignment of trajectories to
particles (Kaiser, 2005; Schweber, 1986). The remaining audience voiced other concerns and largely rejected the Feynman representation as a useful method for simplifying and organizing QED calculations. The official meeting notes essentially ignored Feynman’s contribution, and

He reportedly left the conference “disappointed, even depressed” (Kaiser, 2005, p. 47). According to Kaiser (2005), the story of this disappointing inaugural presentation is well documented, but what remains largely unknown is that the confusion and angst surrounding this method took years to settle.

Recall that Schwinger’s presentation and new methods were both very well received by those at the Poconos conference. Feynman’s techniques only began to take hold once Freeman Dyson was able to demonstrate that, mathematically, Feynman’s methods yielded the same results as Schwinger’s (Goldstein, 2008, personal communication). This work took place following the Poconos conference and paved the way for Feynman and Dyson to take center stage at the subsequent NAS conference, at Oldstone. While Schweber (1986) writes that the
power of Feynman’s work was realized at this conference in 1949, newer accounts written by Kaiser (2005) suggest that the larger theoretical physics community did not fully recognize the merits of Feynman’s work until well into the 1950s. Kaiser (2005) cites examples of conversations throughout that time period where great researchers hinted at the power of the Feynman Diagram, but did not incorporate it into their work. Hans Bethe, someone Feynman had developed a relationship with at Los Alamos during the Manhattan Project, began to coach students on the use of these diagrams while on the faculty at Cornell University, along with Feynman and Dyson. As more and more notable figures, such as Bethe, began promoting the use of these diagrams, they began to take root as conventional methods for completing QED calculations. Ultimately, while the introduction to this representation was poorly received by leading researchers in the late 1940s, the Feynman Diagram revolutionized QED and is considered a cornerstone of theoretical physics to this day.

The historical account of Feynman’s introduction of his diagrams highlights a number of aspects of representation that will be addressed sequentially. A graphical representation of such a complex idea as QED proved to make the concept accessible for application in problems central to the field of physics. While this representation certainly helped physicists think about QED, it also necessitated the development of common understanding or shared meaning. In other words, until the community came to accept the usefulness of this representation, it was not revered as the monumental discovery that it actually was. The process of finding meaning in a new form of representation relates to the idea that symbols used in the external world must be linked to one’s understanding of the concept being represented. However, a distinction must be made between the appropriation of conventional representations and the creation of idiosyncratic externalizations. Spontaneous, idiosyncratic representations produced by children
are born out of some idea in the mind. The link between concept and representation, in this case, has to be present at the time the child generates the externalization. Alternatively, with conventional representations their exists an iterative process of externalizing an idea and re-linking the artifact to the understand that is essential for finding meaning in the conventional elements and systems. Using symbols and representations as means for developing knowledge in such an iterative process has been termed progressive symbolization (Enyedy, 2005), and is a powerful construct in thinking about how representations contribute to the construction of an understanding.

2.3 Progressive Symbolization in Children

When a child writes or draws something, that externalization, we assume, is linked to an understanding in the mind. In other words, the signified is linked to the signifier by some “associative bond” (de Saussure, 1959, p. 66). However, once a representation becomes objectified (e.g., in a written form on paper or an intentional gesture), the link between the conceptual object and the understanding needs to be re-constructed. Such is the case for children adopting conventional systems of representation such as written language or mathematical notation, for example, what a negative sign represents in reference to velocity and motion (Nemirovsky, 1994). Children must construct a link among the conventional notations such as letters and numbers, but they must also continually re-construct the association between whatever idiosyncratic representation they produce with their own understanding as well (Goldin & Kaput, 1996; Lehrer & Schauble, 2002). There is a process of production and realignment
which deeply impacts how children represent their ideas, and the work of diSessa, Hammer, Sherin, and Kopalkowski (1991) on children’s inventing graphing highlights this process.

In a relatively short intervention (five class sessions) with sixth-grade students in Oakland, California, diSessa et al. (1991) demonstrated that children have a natural ability to invent and critique representations. Students were presented with the following scenario:

* A motorist is speeding across the desert, and he’s very thirsty. When he sees a cactus, he stops short to get a drink from it. Then he gets back in his car and drives slowly away. (diSessa et al., 1991, p. 125)

Each student was charged with putting something on paper that described the motion of the motorist. The students’ first attempts at representing the scenario yielded idiosyncratic notations based on lines, dashes, dots, and other relatively generic shapes\(^\text{13}\). Discussions about each notation lead to realizations by the students that more than one dimension needs to be included in a representation of the motorist’s trip. For example, some of the initial drawings only represented changes in speed without consideration of changes in time or distance. Through revisions of their original notations, the students developed more complex notations showing distance and time. In each successive revision, the conversations between the students and the teacher served as a way for each individual to find some meaning in the newly produced form. Each student needed to match the specific version of the representation with his or her understanding of the motorist’s path through the desert. Eventually, the students came to an agreement on a final representation for speed, distance, and time which was a position versus time graph – a conventional method for representing motion. This episode highlights two critical elements of children inventing representations. The first is that through invention and critique of

\[^{13}\text{The term notation is used in reference to markings on paper that students generated in the diSessa et al. (1991) study. Notation (which is a form of external representation) is the term used by diSessa et al. and in the interest of consistency I have chosen to use the term in this section.}\]

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representations, students can develop and appropriate (when mediated by an expert, in this case
the teacher and researchers) conventional means of representing information (see also Enyedy,
2005). The second important element to this episode is my belief that any externalization of
knowledge requires an accompanying connection between the artifact and each individual’s
understanding. Even when the individual produces the representation (as opposed to viewing
one that someone else produced), he or she critiques the artifact, and inherent in that critique is
the creation of an association between the external artifact and the ideas the individual holds
about the particular situation.

Many researchers have shown that children have a powerful ability to critique
representations (diSessa et al., 1991; diSessa & Sherin, 2000), and such evaluations of the
artifact are central to the reconstruction of the link between the object and the signified.
Bamberger (2006) proposed that for musicians and children learning music, “performances
(both silent and out-loud) involve a process of active, sense-making occurring in real-time” (p.
70). While Bamberger’s work refers to music, I believe it is pertinent to the current discussion.
Consider the externalization of knowledge to a be performance, whereby the child is
demonstrating some aspect of knowledge. This notion of active, real-time sense-making
accurately describes how the child produces and reflects upon a representation, as to rebuild the
link between idea and object. A second attempt to reproduce the same representation will likely
reflect this newly formulated association and may result in a more complex or refined
representation. This “second generation” representation undergoes the same re-linking process
described, and this iterative process has been called “progressive symbolization” (Enyedy, 2005;
Lehrer & Schauble, 2002). The question that progressive symbolization attempts to address is
how “the process of progressively refining one’s representation of some aspect of the world can
contribute to a deeper understanding of a domain” (Enyedy, 2005, p. 428). Alternatively, Karmiloff-Smith (1990) proposed *representational redescription* as a similar construct, where students undergo developmental changes whereby they represent elements of the same knowledge in progressively more abstract ways over time. Children regularly engage in such progressive or re-representational activities, particularly in mathematics and science learning, which leads to increasingly complex understandings of the particular domain of study (Lehrer, Strom, & Confrey, 2002). An example is the lengthy process of children coming to appropriate the conventional forms for writing numbers. Although they may begin to use the forms from a very early age (around 3 in most settings in which they are able to interact with these forms), it is only around the age of 10 that children truly and deeply grasp all the nuances underlying the decimal place value written number system. This lengthy process is grounded in continual redescriptions of the written forms to children’s understandings of number and the logic of the system. Underlying this process of change is the notion that children are iteratively or cyclically producing and refining representations of knowledge, and the refinement is heavily influenced by the ability of the child to re-construct a link between object and idea. For example, children inventing methods for mapping height showed a progression from overhead representations to eventually creating and adopting topographical lines to represent changes in height (Enyedy, 2005). Similar to how students re-invent graphing (diSessa et al.), each successive attempt to refine the representations resulted in complimentary conceptual gains such that the new representations matched the understandings of the children. The act of representing and re-representing to refine knowledge (Waldrip, Prain, & Tytler, 2008), and to define the symbols themselves, is analogous to what scientists and mathematicians do in their respective domains.

14 Karmiloff-Smith’s (1992) *representational redescription* model is primarily concerned with internal representations, however, external notations play a role in the construction of such internal representations, thus this construct has relevance.
In a domain such as Theoretical Physics, efforts to generate new models that simplify the representations of concepts are what drive the field. In doing so, scientists utilize conventional systems of representation as well as introduce new elements to better describe the concept, such as with the Feynman Diagram. Children engaging in progressive symbolization go through a similar process. As they objectify knowledge, they will slowly incorporate and appropriate elements from conventional systems of representation. Eventually, children come to learn the conventions and how they are used. The iterative process described is an example of how externalizing knowledge can serve as a thinking tool: by becoming object, it allows us to reflect on them, verify our thoughts, and contrast between different as well as contrast understandings and representations. It also highlights the importance of conventional systems of representation, the origins and development of which are deeply influenced by cultural factors.

2.3 The Role of Culture in Representation

Discussing representation as though it were a simple mechanism, uninfluenced by factors in the environment or body is naïve. In fact, the development of the ability to represent knowledge is shaped by a multitude of factors, not least of which is culture. Piaget and Inhelder (1966/1969) argue that all children develop similar logical structures of mind, regardless of cultural influences or content domains. Their contention is obviously valid, particularly with regard to the role of representation in developing logical thought; however, it is plausible that representation is inextricably linked to the cultural environment within which it is generated.¹⁵

¹⁵ David Henry Feldman (personal communication, October 31, 2007) would argue that attempts to connect Piaget’s and Vygotsky’s theories would fundamentally violate the assumptions of both theories. Feldman would argue that such attempts constitute a misinterpretation of both theories. While I respect this perspective, especially with regard to child development, others have found methods for combining aspects of each theory in education. Paul Cobb
For example, Lucienne’s first attempts at representation centered on a matchbox and a watch chain. The physical action she represented is likely culturally independent; however, the objects she interacted with certainly have cultural significance. Lucienne’s gesture was directed toward a matchbox and chain, which on some level are associated with a particular cultural context; thus, her representation was necessarily influenced by culture\textsuperscript{16}. Perhaps a more powerful example of the role of culture in representation is the case of Jackson, Stephen, and James and their descriptions of the ways in which parachutes work. Their introduction of ideas like x-ray, payment for services, and written English language are all products of culture and they played a key component in how these students expressed their understandings. James used what he knew about x-rays (the x-ray “sees into things”) as a vehicle for representing the activity inside the parachute. He also used arrows to indicate movement, a classic representational tool in the sciences, particularly in physics. The use of culturally-developed representations to not only express knowledge of a situation, but also the science that explains (or at least attempts to explain) that situation is undeniable evidence that children incorporate cultural ideas and artifacts into their externalizations. Whatever the culture may be, individuals belonging to this group will develop means of expression which embody shared-meaning and become conventional. In other words, a desire to communicate will drive a culture to invent ways of talking, writing, or even gesturing that all members can use and understand. One could make the case that all

(Cobb, Yackel, & Wood, 1992; McClain & Cobb, 2001; Yackel, Cobb, & Wood, 1991) writes about a social constructivist perspective, which honors the constructivist nature of Piaget while valuing the role of the teacher from a Vygotskian perspective. Hatano (1993) has similarly found ways of reconciling constructivist notions with Vygotskian views on knowledge acquisition.\textsuperscript{16} Culture can be defined in numerous ways. For example, countries have their own cultures (e.g., British culture), regions can have their own cultures (e.g., Western culture), and domains can have their own culture (e.g., the culture of science). All of these variations in definition can impact representation, for the individual frequently crosses cultural boundaries depending on the context or activity he or she engages in. In school, the child is part of a particular academic culture; however, in science class s/he is also engaging in the culture of science. Thus, in this paper, the term culture is purposefully vague as to illustrate that all cultures influence an individual’s representations.
representations embody, on some level, aspects of their native culture because they serve as vehicles for interactions among individuals (Cole, 1996; Nelson, in press; Vygotsky, 1978). Language is, perhaps, the prime example of a cultural invention that deeply impacts the ability to represent an understanding; for language is what opens the doors for many children to begin experimenting with externalizations.

When children begin developing the ability to represent ideas, through language or any other form (i.e., gesturing or drawing), they are doing so within a cultural context laden with conventional systems. And while learners may not master the conventions, these cultural standards influence how children spontaneously represent knowledge at younger ages and how they eventually come to master and appropriate (granted, each individual with his or her own particular signature and flavor) a conventional system such as written or oral language, or the written number system. The conventional systems serve as a gateway for learners to begin to understand the more complex and abstract conceptions that are prevalent in their particular culture. While culture shapes representations, it also shapes the use of these tools and the nature of knowledge. Conventional systems are developed by cultures as they negotiate a shared meaning about a particular concept (Confrey, 1991). Convergence of meaning amongst members of a culture (i.e., engineers or physicists) results in such negotiations which center on the external representations of the concepts in question (Confrey, 1991; Roth & McGinn, 1998). Integrated within these negotiations are decisions and agreement about what symbols are to be used to represent which conventional ideas. Thus, those ideas which are accepted by a particular culture may be added to their collection of conventionalities. In fact, representation systems such as written language and the mathematical notation systems are products of human attempts to describe and share meaning over centuries of development. Thus, the role of social interaction
and negotiation plays a crucial role in how conventional representations come to be, as well as in how students come to know and use these systems in order to express their ideas.

### 2.4 Culture and Representation in Science: Copernicus’s Proposal

There are a few crucial points in the history of science where substantial discoveries altered the path of scientific investigation. Such discoveries achieved elite status in part because the representations of these ideas were debated and eventually accepted by the scientific community. Nicolaus Copernicus’s proposal that the universe revolved around the Sun was just one of those moments of discovery. Often regarded as “the major contribution to scientific thought” (Gribbin, 2002, p. 8), Copernicus developed a new representation for the universe that comprised a radical shift from the widely accepted Earth-centered representation of the time. Copernicus waited to publish his model until late in his life largely because he felt it did not answer many of the questions unanswered by the previous models (Gribbin, 2002). Nonetheless, when his masterwork *De Revolutionibus Orbium Coelestium* (*On the Revolution of the Celestial Spheres*) was published in 1543, very few scientists of the time grasped the implications of this proposal. Copernicus’s representation of the universe was so unfamiliar, that it took a great deal of time (long after his death, also in 1543) before it became accepted as a new model of the universe. Had Copernicus lived to promote the ideas of his model, perhaps a more storied debate around the proposed idea would have occurred. Regardless, the length of time and relatively slow adoption of this idea shows the role of socially-accepted meaning in representation. One cannot simply put forth a new conception and representation and expect the world to accept it as truth. As with all ideas, a negotiation of meaning helps to define the
concept, and that negotiation is dependent on the representations of the proposed idea. Copernicus’s model is now accepted as scientific doctrine, backed up by many research studies that provide irrefutable evidence to support his claim. In essence, his proposal became conventional wisdom for the scientific community through a process of social negotiation resulting in shared-meaning.

2.5 Representations as the Centerpiece of Negotiations about Scientific Meaning

From a more philosophical perspective, it can be argued that a concept as we know it (e.g., gravity) does not exist without its accompanying representations. Plato (1963) offered that on another plane of existence, inaccessible to mere mortals, there lie a set of ultimate truths about the world. I dismiss this perspective for the genesis of knowledge and adopt an alternative view which purports that the scientific concepts humans have agreed upon to be “true” are dependent on the representations of these concepts, because these representations serve as the focal point for the negotiations that lead to accepted meaning. When a scientist discovers a new phenomena (which I acknowledge can impact the world before a representation of that concept has been developed), he or she constructs a means for communicating this new idea to the scientific community. For the domain expert (i.e., scientist), such a representation will likely include elements from conventional systems such as graphs or mathematical notation. Once the idea is objectified in the artifact, conversations regarding the validity, robustness, and accuracy of the proposed concept eventually decide its fate. For some, the representation chosen by the discoverer may not be suitable and, thus, may be modified to satisfy concerns. Others may accept the representation but challenge the methodology through which the evidence for such a
phenomenon was generated. For example, when Einstein proposed his special theory of relativity, which revealed the equivalence of mass and energy with the famed $E=mc^2$ algebraic representation, it was of limited impact on the field of physics (Gribbin, 2002). It is inaccurate to say that his contemporaries did not understand his theory, but the ideas did not make a significant impact until translated into four-dimensional geometric terms (Gribbin, 2002). At the time, algebraic notation and geometric notation were both conventional in science; however, it took some negotiation and discussion before the field came to unanimously accept the theory as accurate and powerful. At the center of the discussions around newly introduced concepts is an externalization of some knowledge, and eventually the community will come to either reject the idea or find shared-meaning in the representation which would elevate the idea to a level of consensual knowledge (Kaput, 1991). We need to call attention to the inherent pattern existing within this negotiation of meaning (Confrey, 1991). Progressive symbolization, as described above from the standpoint of the individual child, can now be expanded to include the role of social interactions in learning to represent. For the community, the external representation helps to organize and refine a shared understanding, while each individual must reconstruct his or her association between the representation and the underlying concept. Therefore, the inherent pattern found in instances of meaning arriving from negotiation is present within the individual and within cultural contexts.

2.6 Social Aspects of Representational Development

The development and evolution of conventional systems of representation was primarily driven by humans’ desire to communicate thought (Donald, 1991; Olson, 1994). As individuals
invented and incorporated new elements into these systems, they were either useful in communicating ideas or not; the same situation occurs when new representations are introduced into modern systems. If a new element is found meaningful, it will be incorporated into the new system either based on existing rules or it will require the creation of newer rules. New elements become conventional through the process of groups of people finding meaning in the particular usage with which they are employed. Stated differently, conventionality is evidence of shared meaning in use. With language, for example, the development of syntactical and semantic rules was the product of a need for formalizing the communicative tool such that meaning could be retained across contexts and over geographical distances. Moreover, conventional systems of representation hold within them a record of the historical development of meaning for the particular culture that employs this system. When children in modern times begin to develop the semiotic function and learn tools for communication, they do so immersed in an environment where conventional means of expression are omnipresent. Students learn to appropriate and develop representations that have inherent elements of conventionality. In doing so, they engage with the shared meaning that has developed over centuries through social interactions performed by members of a culture. The child learns to find shared meaning in representations with other members of the culture, in a similar way to how conventions developed over time. For example, while Feynman was not a child at the time he introduced the diagram, the process of finding shared meaning is quite similar. The members of the particular QED culture at the Poconos meeting did not find meaning in the newly formed representations. It was some time before they saw this as a useful tool, and only then did they accept the diagram which has become a conventional method of representing QED concepts.
Humans are inherently social organisms, and children hold strong desires to communicate ideas with other individuals in their world. The referential-communicative aspects of representation inspire children to develop increasingly conventional methods of representation, as these conventional methods are what grant them access to a more complex level of interaction with other members of the social world. The signs and rules of particular conventions are what the child must come to learn in order to become an effective communicator. From a young age, children engage with these representational objects before developing the ability to appropriately use them.

Obviously young children do not understand what these notations refer to or what they are used for. None the less, it is possible that even before they fully understand these notations, young children are sensitive to the formal differences between drawing, writing, and numerals. (Tolchinsky-Landsmann & Karmiloff-Smith, 1992, p. 288).

Children’s awareness of the differences between systems of conventional representation suggests that there is a social dimension in how they come to learn and appropriate representations, one aspect of which is the role of feedback in learning to represent.

The child begins to represent knowledge in idiosyncratic ways, all the while gathering feedback from others (in the form of spoken words, gestures, facial expressions, and other means) which helps the child to determine the effectiveness with which he or she externalized an idea. When a child utters a phrase, he or she perceives how that phrase was received by others in similar ways to progressive symbolization. The phrase may be repeated or the conversation may be extended as the child iteratively communicates an idea. Bakhtin (1986) emphasizes this point with his belief of the role of spoken language in helping individuals refine ideas. The feedback cycle built into spoken language communication is an example of how external representations can
act as tools for refining understanding (and opinion for that matter). This social dimension of representation is where children begin to adopt and use conventional signs and means of expression. Thus, it is important to consider referential-communicative attributes in research on how children come to represent understanding in any domain.

It is evident, as the previous sections have shown, that representation in science is a valuable and meaningful lens through which to analyze how children come to understand scientific ideas. The variety of ways in which representation impacts understanding and is part of understanding make it a powerful construct for research. Additionally, the dependence on multiple systems of representation in the practice of scientific fields (including mathematics, engineering, and technology development) is further evidence that children should be led to focus on representation in the learning of normative concepts. Further definitions of the components of representations as a research construct are needed in building the argument for this work.

3.0 WHAT IS REPRESENTATION?

The term representation is heavily used in education, child development, and psychology literature. The definitions offered in the introduction to this paper only graze the surface of the complexity of representation as a domain of scientific investigation. This section seeks to organize the ways in which representation has been addressed in the literature, particularly with regard to work in mathematics and science education. I will culminate with an articulated position for how I believe representation should be considered in the field of science education.
3.1 The “Stand-for” Relationship

Much of the early work on representation stems from linguistics and attempts to unpack the complex relationships between sign, signified, and signifier (de Saussure, 1959). At the most basic level, representation has been treated as a “stand for” (Goldin & Shteingold, 2001; Kaput, 1998; Lee & Karmiloff-Smith, 1996b; von Glasersfeld, 1987) or “corresponds to” (Kaput, 1998) relationship between one part of an individual’s experience and another. Take de Saussure’s (1959) example of arbor. He differentiates between a sign, signified, and signifier using the following pictorial example (see Figure 7):

![Figure 7. de Saussurre’s (1959) image showing how each element is intimately related in both directions (indicated by the arrows).](image)

The written word arbor is called a sign by de Saussure because it carries the concept of “tree” and what he calls the “sound-image” (meant to encompass both the spoken and the written versions of the word). The graphic image of a tree would, then, be the signifier, also considered a sound-image. The concept of “tree” remains as the signified in each case. de Saussure continues to deconstruct the signifier into sign and symbol (similarly to how Piaget used the terms), whereby symbols are the motivated signifiers such as the tree image above, and signs are conventional, arbitrary signifiers. “Every means of expression used in society is based, in principle, on collective behavior or – what amounts to the same thing – on convention,” (de

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17 More contemporary work on written language, such as Ferreiro, points out that de Saussure paved the way for ignoring the written aspects of language by encompassing them within the oral aspects (see Ferreiro, 1994; Ferreiro & Teberosky, 1979).
Saussure, 1959, p. 68). The English written letter system provides for the word *arbor*, which is arbitrary in that the graphemes are not immediately associated with the image of a tree, and these conventional signifiers are called signs. de Saussure’s discussion of the nature of the linguistic sign (i.e., representation) highlights a few key aspects of representation on which I will expound.

While there is some general agreement that representations establish “stand-for” relationships (Kaput, 1998), the spoken, written, drawn, and even gestured entities can carry different meanings. An adult, presumably operating with a sufficient knowledge of a conventional system of representation, continually varies the meaning of representations depending on context and situation (von Glasersfeld, 1987). Thus, while there are conventional systems, there are not always conventional meanings. Some have a tendency to believe that words and images have an embodied meaning that exists as such, without interpretation. While this may be true on some level and for individuals with command over a conventional system, children engaged in the process of learning to create “stand-for” relationships are unaware of these pre-determined relationships between signifier and signified. As von Glasersfeld (1987) states:

Because perceiving, from a constructivist point of view, is always an active making, rather than a passive receiving, the similarity of a picture and what it depicts does not reside in the two objects but in the activities of the experiencer who perceives them. Ordinary language, however, refers to objects as though they existed as such, independent of experience. Consequently, it always leads us, the language users, to attribute differences in our perceptual operating to the externalized objects as though there were properties belonging to them in an “objective” sense. Provided we remain aware of this epistemological sleight of hand, we may safely say: An iconic representation...is an
artifact and a deliberate reconstruction of another experiential item; the reconstruction selects certain properties considered relevant under the circumstances. (p. 217)

Therefore, in defining representation with regard to children, it is important to consider representation as a useful referential-communicative tool (Tolchinsky-Landsmann & Karmiloff-Smith, 1992); this would constitute the process of representing, as discussed earlier. de Saussure’s example also highlights a controversial dualism in representation research, which is closely linked with the discussions of the process view of representation. This dualism is the distinction between internal and external representations.

3.2 Internal vs. External Representations

Literature reporting traditional approaches to cognitive research has regarded representations as existing exclusively in “the mind” (Zhang & Norman, 1994). These internal representations are the subject of a large body of literature on knowledge organization in the mind, also referred to as students' internal conceptualizations (Lesh, Post, & Behr, 1987) or mental representations (Brizuela & Earnest, 2007). Donald (1991) suggests that as cultures have developed, they have relied more on external memory media, devices such as language and written notation systems that offload cognition onto the external world, subsequently freeing working memory for use in more complex tasks. Others have supported Donald by focusing on external representations (Even, 1998; Martí & Pozo, 2000). According to Donald (1991), external representations of memory are mechanisms of cultural evolution and of the development of the modern human mind. Goldin (1998) defines external representations as "the shared, somewhat standardized representational systems developed through human social processes" (p.
supporting the belief that cultural evolution plays a critical role in the development of these systems. However, before the issue of systems of representation can be fully explored, the distinction between internal and external representations must be addressed. Goldin and Kaput (1996) state that, “the distinction that we make between external and internal systems of representation is itself simply a constructed model, developed by an observer or community of theorists to help explain an individual’s observed behavior, or the behavior of a population of individuals” (p. 407). This “phantom of dualism” (Pérez Echeverría & Scheuer, in press) that the internal/external distinction evokes fuels the debate over whether there can exist such a separation. Defining these concepts independently is consistent as a classification scheme for the purpose of research, therefore, I am not critical of Goldin (1998), Goldin and Kaput (1996), or Martí and Pozo (2000). However, one must acknowledge the relationship between internal and external forms of representation in order to avoid false implications of duality.

Pérez Echeverría and Scheuer (in press) raise the issue of whether a focus on external representations is “establishing an absolute frontier between outer and inner worlds” (p. 7). Discussions of mental representation have a danger of suggesting that mapping perceptions of artifacts that exist in the external world onto the internal mind is simply a collection of reproduced images, written notations, colors, sounds, or even gestures. To imagine that a child sees an image and generates a carbon copy of that image in the mind ignores the complexity of the relationship between the external and internal. Discussions whereby this literal mapping scheme is evoked have a tendency to emphasize the “phantom of dualism” because they downplay the role of sensory experiences in perceiving information, which some have warned is a dangerous trend in representations research (Nemirovsky, in press; Pérez Echeverría & Scheuer, in press). Nemirovsky (in press) writes,
Seeing an image is not a matter of forming a neuronal “version” of it, but of an activity that engages the motion of our eyes and bodies, the detection of edges and colors, the stereoscopy discrimination between the two eyes, and so on. Our experience of seeing something emerges from all the activities that incorporate it – *incorporate* in the sense of our body shaping itself to make room for the image. The bodily activity of seeing is not a constitution of representations, in the same sense that our bodily activity of eating is not a matter of constituting representations of food. (p. 1-2).

As previously mentioned, the act of externalizing understanding involves a remapping of that conceptual object to the understanding one has. Such a process of remapping is exactly what Nemirovsky (in press) speaks toward; children do not copy what they perceive onto their mind; rather, they reconstruct connections between perception and understanding. Thus, when the focus is placed on internal versus external representations, the constant interplay between understanding and externalizations is essentially ignored. As some researchers have suggested (Nemirovsky, in press; Pérez Echeverría & Scheuer, in press), the focus needs to be placed on the inscriptions, notations, and symbolic expressions that children produce in particular learning environments.

The current review is interested in externally produced expressions with a physical presence, that are typically referred to as notations or inscriptions in mathematics education (Brizuela & Earnest, 2007; Goldin & Shteingold, 2001; Lee & Karmiloff-Smith, 1996b; Lehrer & Schauble, 2002). Therefore, it is important to remember that even though in this review I may focus on children's externally produced representations, I certainly acknowledge the lurking presence of “the phantom of duality.” In working with children in educational settings, the only primary source of their understanding that we, as researchers, have access to are the notations,
inscriptions, drawings, gestures, and speech that they produce. Each of these modes of expression can be organized into a system of representation (Gardner & Wolf, 1983; Goldin & Shteingold, 2001; Nemirovsky, 1994), which are useful constructs for considering how children learn and appropriate representations.

I must briefly comment on the diversity of opinions regarding terminology around external representations. While some researchers and theorists intentionally avoid the term *symbol* (Lee & Karmiloff-Smith, 1996b), others actually prefer to use the term symbol (Gardner & Wolf, 1983; Nemirovsky, 1994). As mentioned above, mathematics education researchers have primarily used the terms *notation* and *inscription*, however, these classifications fail to include gesture as a legitimate form of external representation, which some researchers believe are just as important as those representations existing on paper (Goldin-Meadow, 2003; Noble, 2003). At the heart of these distinctions is the question of whether one views a representation as existing independently from the person or from time (Lee & Karmiloff-Smith, 1996b). For example, spoken language or gestures, unless recorded, only exist in the moment, while written language exists independently from time. I choose to address these classification terms through the lens of systems of representation, which consist of elements (e.g., letters and numbers) and rules governing the use of such elements.

### 3.4 Systems of Representation

Modes of expression that are conventionally used as referential-communicative tools must meet two central conditions: they must have elements (e.g., signs/symbols, inscriptions, notations) and they must have rules which govern the relationships between elements. A single
notation or inscription is essentially a meaningless signifier unless it is situated within a larger *system of representation* (Gardner & Wolf, 1983; Goldin, 1998; Goldin & Shteingold, 2001). Goldin (1998) cites “systems of spoken symbols, written symbol, static figural models or pictures, manipulative models and real world situations” (p. 143) as examples of systems of representation. Nemirovsky (1994) differentiates "symbol use" from "symbol systems" by maintaining that systems involve rules. Recall the role of cultural evolution in the development of representations. As conventional systems of representations used as referential-communicative tools are created, rules are simultaneously developed (either implicitly or explicitly) as means for preserving common meaning found in the relationships between signifier and the signified. Thus, the rules which govern the use of elements evolve so that the arbitrary signs of a given system may be consistently mapped to intended meanings.

Agreement that systems of representations are dependent on rules provides the foundation for a richer discussion as to the variety of dimensions that can be used to classify systems. Nemirovsky (1994) parallels the discussion of symbol-use versus symbol-systems with Bakhtin’s differentiation between sentences and utterance. In Bakhtin’s (1986) discussion of a sentence, he comments on how written language is not merely a copy of spoken language (evoking Ferreiro’s point about de Saussure’s focus on orality), because spoken language is delivered with affect that is limited in written language. A sentence follows the rules of grammar but leaves out a wealth of information, such as intention, that is conveyed with expressive intonation. As Nemiroksy (1994) says, “The actual manifestation of these communicative intentions cannot be reduced to a formal analysis based on syntactical or semantic rules; one has to consider gesture, what had been said before, facial expressions, who is talking to whom, etc.” (p. 390). A written sentence, by adhering to semantic and syntactical rules,
conveys different information than does the utterance. The child’s use of a symbol in an idiosyncratic manner is akin to the utterance, while more formal representations generated with knowledge of a rule system are akin to the sentence. The reason for making this distinction is to highlight the variety of ways in which a system can be used and defined. In both the sentence and the utterance, the entities used are signs from the same collection of signs (i.e., words), which constitute the language. Recall the case of invented graphing (diSessa et al., 1991), described earlier. One particular student, Mitchell, chose to represent speed using slanted lines. He argued, “If the line is horizontal, he’s going really really fast. And the further up the line slants, the slower it goes. And then when it gets like this (vertical), it is a stop” (diSessa et al., p. 131). Mitchell’s slanted lines are similar to the sloped lines used in conventional graphing techniques for speed. However, his usage was not specifically related to a mathematical slope, thus he was using a conventional element (i.e., the sloped line) without necessarily adhering to the conventional ways in which those elements are used. Mitchell was not aware of the rules of the graphing system of representation, but he managed to effectively use the elements of that system to begin expressing an idea. He begins to find meaning in the rules of the graphing system through repeated discussion (as the study shows) and re-representation in varying contexts. Moreover, Mitchell’s first interactions with the graphing systems are from a process perspective, and once the rules of the system become apparent and understood, he can begin to treat graphs as conceptual objects in their own right.

When children learn to represent, they must learn not only the notations and inscriptions for elements within the system (e.g., words and numbers), but eventually the rules of the system as well. However, not all systems have formal rules. For example, when two people speak colloquially, there may not be an agreed upon set of rules for that system of spoken language, but
they are still able to represent meaning. Lehrer and Schauble (2000a) use the term "representational model" to refer to such representations that can be part of a system but can also be unconventional and unsystematic (Brizuela, 2004). Thus, a need arises for a classification scheme for systems of representation with similar rule systems, as this will help in classifying how students learn to represent knowledge. Pérez Echeverría and Scheuer (in press) offer an interesting classification scheme for external representations with four major kinds of external representations: (1) bodily and gestural representations, (2) oral language, (3) notations based on relatively relaxed combination rules (i.e., drawing, maps, graphs, and slang speech), and (4) strict notational systems18 (Pérez Echeverría & Scheuer, in press). According to this taxonomy, there can be systems of representation with formal rules and different systems built on relaxed, idiosyncratic rules. The authors purport a second axis of dimensionality to the classification scheme, which is a temporal – spatial scale. Consider gesture and oral language as two systems of representation, whereby gesture has largely idiosyncratic rules and oral language is governed by much more formalized and conventional rules (i.e., grammar). As Pérez Echeverría and Scheuer (in press) suggest, the physical distance between the individual producing the representation and the individual receiving the representation widens as one moves from gesture to oral language. In order to interpret a gesture, physical proximity and visual contact between the producer and receviever must occur. However, oral language communication can occur without visual contact with one another, and given the introduction of phone or internet technologies into communication, oral conversations can take place over great distances19.

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18 The authors acknowledge that there are additional forms of external representation left aside, including deaf sign language, reading and writing Braille, video and film, written records of choreographies, and combined use of several external representations in computerized environments (Pérez Echeverría & Scheuer, in press).
19 Technology has complicated the ability to characterize systems of representations in this way as recording devices have become relatively ubiquitous. Gesture can be conveyed over distances using video conferencing equipment or even video recordings. However, the classification scheme remains useful in attempting to make sense of how children come to operate in these numerous systems of representation.
While physical proximity is an important factor between oral and gestural representations, those externalizations which produce a physical artifact (e.g., notations, inscriptions, etc.) can exist over a longer temporal dimension as well. Representations of ideas which yield a physical artifact are perceivable over long periods of time, across contexts, and across cultures. The lasting nature of such representations may make them more useful in reflective thought or in aiding memory (Pérez Echeverría & Scheuer, in press). Thus, systems of external representation appear to have both temporal and spatial qualities, which offer different benefits in different contexts. Analyzing systems of representation using such a construct, where the structure of a rule system comprises one dimension of the system and the temporal–spatial constraints comprise the other leaves the impression that these systems are separable. While they may be separable in terms of research, “they continually coalesce in everyday activities” (Pérez Echeverría & Scheuer, in press, p. 6). The important overarching idea to remember when discussing systems of representation is that each system contains elements and rules, and children must learn both aspects en route to becoming proficient within each system.

Brizuela (1997) shows that failure to adhere to the rules of a system does not necessarily detract from the meaning of the representation in the child's mind. This is an important point to keep in mind while researching how children produce external representations. While some of the artifacts may appear meaningless from the perspective of a developed system of representation, in reality the artifact has meaning to the child, and this meaning should be explored. Therefore, researchers must value how children represent spontaneously, how they learn to represent, and how they come to learn the common rules associated with specific systems of representation, be it formal or informal. The example of Mitchell from diSessa et al.
(1991) put forth above is a prime example of importance of valuing spontaneous representations.

4.0 **Evolution of Representation**

Piaget, and others (Carey, 1991; Carey, Wiser, & Smith, 1985) have studied the evolution of scientific concepts throughout history and compared these with children’s construction of scientific understanding. In doing so, Piaget found that as children construct knowledge of the natural world, their trajectory is marked by certain conceptual stumbling blocks that tend to mirror the historical trajectory of major scientific discoveries. The same concepts that puzzled the great scientists of the past tend to also puzzle children. The comparison does not presume that children re-create major scientific discoveries completely on their own; rather, those ideas which humankind struggled to understand or represent are the same ideas with which children will have trouble comprehending because they are conceptual or epistemological stumbling blocks and obstacles. Comparing the path of a child’s scientific learning with that of history helps researchers better understand how children come to see the natural world. A similar comparison can be made between the historical development of systems of representation, such as writing or the written number system, and the conceptual evolution children follow while learning to represent knowledge. For many of the reasons already outlined in the previous sections, there are parallels between how conventional systems of representation developed and how children come to operate within these systems. Thus, recounting the evolution of notation into the variety of systems used in scientific practice and learning today is central to the overall questions of this review.
Humankind’s development of the desire and ability to record speech, which preceded written notation, is debated by numerous scholars. A popular historical belief was to regard writing as the physical instantiation of spoken words; the intent of writing being to record speech. Olson (1994) refutes this point and argues that writing systems developed not to record the sounds of speech, but to communicate information; the relationship with speech is indirect (also Ferreiro’s (1994) point). Rather than acting as a recording instrument, writing serves as a model for speech (as will be illustrated by the Three Sheep example). Olson says, “Our intellectual debt to our scripts for those aspects of linguistic structure for which they do provide a model and about which they permit us to think, is enormous” (Olson, 1994, p. 89). The evolution of said linguistic structures, which developed concurrently with written notation, provided for the creation of culturally-specific language and an alphabet, which are both examples of conventional systems of representation born out of humans’ attempts to put the world on paper (Goldin & Kaput, 1996; Olson, 1994). In much the same way that humans’ attempts to record life, efforts to record science have been part of this same representational evolution. Science is built upon written language and written number, which justifies focusing on the story of how these systems came to be. An evolutionary perspective of representation also holds the potential to highlight crucial features of representation that must be brought to the forefront of discussions about how children externalize understanding. Prior to the development of speech or writing, the major evolutionary achievement for homo sapiens was a capacity for mimetic representation (Donald, 1991). Mimesis requires “a degree of social attribution, some skill at pedagogy, and both social coordination and collective knowledge” (Donald, 1991, p. 217). Communication is equally as dependent on social interaction, which is how mimicry developed into speech. Infants demonstrate the ability to imitate at the beginning of their life.
acquisition of the semiotic function (Piaget & Inhelder, 1966/1969), and that is only one dimension of the parallel between humankind’s development of language and that of children. Before discussing the similarities between how children learn to represent and how humans developed that capacity, I propose a framework based on the trajectory of the creation of written language to frame the discussion of how children acquire representational ability. According to Olson (1994), however, the trajectory of the creation of written language involves initial developments of written number. Considering this interplay from a scientific representation standpoint makes the connection logical, as the written number and written language systems are fundamental to representations of knowledge for both children and practicing scientists. It also highlights the arbitrary distinctions between areas of knowledge (language and number) and provides a grounding for understanding children’s initial lack of distinction between both (Lee & Karmiloff-Smith, 1996b).

Once humans developed speech, which, like all forms of symbolic devices, is truly an invention (Donald, 1991), they began to use notation. Kaput, Blanton, and Moreno (2007) recount human’s desire to construct generalizations as the impetus for developing symbolization in mathematics. They argue that attempts to express ideas to oneself and to others lead the need for such symbolizations and eventually, in mathematics, symbols served as the means for expressing generalizations. With either written language or numerical notation, the motivation for developing external inscriptions grew out of a desire to express increasingly complex ideas. The developmental progression of written notation began with graphemic depictions of objects, followed by word-based and sound-based syllabic systems, and eventually arrived at alphabets with syntax (Olson, 1994). The evolution of representation into the alphabet or place value notation is what Neugebauer (1951) calls the “most fertile inventions of humanity” (p. 5).
Olson’s (1994) central argument is that the attempts to put ideas and actions of the world on paper are what allowed languages to develop. Neugebauer seemingly agrees with Olson that as notations evolved through efforts to put the world on paper, inventions such as the alphabet and place value are evidence that developing external representations impacted the very knowledge and understanding humans were trying to express.

The earliest evidence of writing are graphic images that surfaced during the Neolithic revolution in the form of drawings and tallies (Goody, 1987). Olson (1994) remarks that while both forms are graphic, historically drawing and tallying have tended to diverge; drawing has remained iconic in contemporary writing systems and tallying has developed into relatively arbitrary and conventional systems such as the written number system. The earliest uses of these writings were to communicate information, and such graphics were “read” in a much different sense than we use the term “read” in modern times (Olson, 1994). In the early graphic systems, “reading” was more like a description of a picture rather than “reading a text.” Graphic images of this sort bring cultural interpretation and meaning into memory as the image has physical likeness to the object. For example, a drawing of a sheep evokes the concept of a sheep, but also that of a four-legged animal, a food source, and a possible source of clothing. In essence, the graphic form (i.e., the image) is a property of the object much like a name, rather than a word, which is a linguistic unit and generally arbitrary. The lack of distinction between name and word is what Olson (1994) considers to be the major hurdle that graphic systems faced en route to developing language. According to him, the introduction of counting and tallying is where writing began to create the distinction between word and name. Tallying imposes a structure onto notations because it requires a count, which is in essence a manner of syntax, and the

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21 A written letter word is an arbitrary element in that the letters themselves bear no resemblance to the artifact or idea being represented (de Saussure, 1959). It is not to say that words are meaningless, rather that words have developed meaning through humans’ development of graphemic and phonemic structure.
development of syntax is perhaps the best example of how written language can serve as a model for speech.

Olson (1994) reports of an example used by Harris (1986) that accounts for the creation of syntax based on written records for tallying systems.

A system which represents *three sheep* by three symbols for a sheep (i.e., sheep, sheep, sheep) is categorically different…from one which represents the same three sheep by two tokens, one representing sheep, the other the number. Just as syntax is what makes a language a language, it is the syntax which makes a graphic system “generative” for it permits the combination and recombination of symbols to express a broad range of meanings. (Olson, 1994, p. 73)

The representation of the integer *three* in this example marks the invention of syntactical writing, which likely paved the way for the development of the concept of abstract number (Olson, 1994). In fact this type of example is used within the written number system as an example of the origin of the introduction of signs for numbers. As syntax was brought into consciousness, word-images were developed (where the sign for “sheep” using characters, rather than a drawing of the animal, denotes a sheep, not the word “sheep”) which eventually gave rise to realization of words as different than things or names of things; words became things (Olson, 1994). Interestingly, children appear to follow a similar progression of symbol use and understanding.

Children acculturating to the sign practices of their community follow a similar progression of graphic symbol to conventional notation (Munn, 1998) as described by the history of written notation. Young children’s acquisition of conventional numerical notation begins with the use of idiosyncratic markings to denote quantity (Hughes, 1986; Munn, 1998). This is followed by pictographic markings, tallies, iconically used numerals, and conventionally used
numerals (Munn, 1998). While the idiosyncratic and pictographic notations often have no recognizable resemblance (they may well have resemblance for the child) to the objects being counted, the use of such graphic representations mirrors that of the early graphic writing systems. As children link the concept of number with the conventional numeral, they are following a similar path as humans’ evolution of notation systems, where conventional signs were invented and agreed upon by specific communities. Lee and Karmiloff-Smith (1996b) provide evidence of children developing the ability to differentiate between drawing and written number or letter systems. When asked to “write a letter to a friend,” preliterate/prenumerate children tend to produce drawings, however, in sorting tasks the same children make clear distinctions between drawing and written letters or numerals. Lee and Karmiloff-Smith (1996b) argue that children are demonstrating their knowledge of differences between the systems, but also knowledge of the different intents with which systems are employed. If the intent of the activity is to communicate information, children may tend toward drawing, whereas with sorting tasks the children are focused on the notations as domains of knowledge. This work suggests that children follow a similar path of development as the evolution of notations in that they slowly develop different uses for different systems while learning about syntax, rules, and conventions. Written notation, according to Olson (1994), is what helps cultures structure speech by developing syntax and conventional practices; written notation also helps children understand number (Munn, 1998). The similarity between children’s development of the ability to use conventional systems of numerical representation with the evolution of written notation throughout history alerts researchers to the need for sensitivity with regard to how children come to adopt and use conventions.
An important difference, however, must be elaborated between the historical development of conventional systems of notation and the development of children’s ability to use conventional elements. As previously stated, convention arises from an invention which takes on shared meaning and usefulness by a community (Donald, 1991; Munn, 1998). The invention of the Arabic numeral system became invaluable to commerce and trade, therefore conventionality was born from usage and shared meaning. Children, however, do not create conventional elements; rather they come to adopt them for usage in particular situations. Alphabets are the signs of conventional written language systems and numerals are the signs of mathematical systems; science incorporates both these systems and more. Yet, in spite of appropriating these conventional and shared systems, each individual user still overlays conventional uses with their individual meanings and understandings. That is, the fact that there is a one to one correspondence in the external versions of the systems used does not mean that there is a one to one correspondence in the users’ understandings and specific uses of the systems. Recall that the language of science consists of multiple representations, thus, the arguments for why the evolution of representation systems like written letters and numbers is important for children’s development of representational competence also applies to representation in science. Recounting the historical developments of once novel and now conventional systems of representation, particularly as they pertain to science, illuminates the conceptual hurdles that may also pose problems for children when they try to use cultural, conventional elements to explain how the world works. It is important to also consider that, as with the Feynman diagrams, the process of developing new conventions and systems of representation is ongoing. The newly invented systems likely also go through some measure of evolution. Given this developmental view of representation, studying how learners' idiosyncratic
systems gradually approximate conventional systems throughout life in order to participate in the cultures of particular domains is crucial to science education. Research on how students represent across systems suggests that they have a tendency to reconstruct, reinvent, and appropriate conventional elements in ways that mimic the development of these conventions throughout history.

5.0 Learning and Appropriating Representations

The environment within which children and adults live and operate is inundated with representations in numerous forms, such as written language, gestures, written numerals, verbal language, pictorial signs and symbols, mathematical notations, graphical representations, maps, and so on. From a young age, children interact with a wealth of cultural representations, such as those mentioned, and the frequency of interactions increases with age (Lee & Karmiloff-Smith, 1996a, 1996b). Exposure to these cultural entities begins at birth and continues as the child begins the process of exploring and learning how to use symbols. The nature of a child’s interactions with elements of representation systems is mediated by adults who have, in turn, appropriated them. And vise versa: children’s interactions with the adults are often times mediated by these tools (sharing a book, speaking to each other, writing letters to each other). By adulthood, most individuals have become comfortable with numerous systems of representation, including verbal language, written language, mathematical notation, and mapping. Scientists, specifically, engage in the normative practice of communicating concepts and discussing ideas through the simultaneous use of multiple systems of representation. Scientific concepts are not accurately or effectively conveyed through only a single system of representation, such as verbal language. It is not only science that relies on multiple modes of
expression, as an indicator of mastery in any domain is measured by the ability to represent the same levels of understanding through multiple systems (Brizuela & Earnest, 2007; Goldin, 1998). Since the “language” that scientists use is intrinsically multi-representational (García-Mila, Anderson, & Rojo, in press; Larkin & Simon, 1987; Pozo & Lorenzo, in press), research into how students learn multiple representations and how they come to apply them is of importance to the science education community. More specifically, analyzing children’s attempts to operate across multiple systems of representation may highlight aspects of representational competency otherwise left unidentified if studied through the lens of a single system. Therefore, in light of the rationale for studying how students come to represent knowledge, a specific focus will be placed in this section on work done in multiple representations of ideas about science.

On a macro-scale, science provides systematic and interpretable ways of describing the phenomena experienced in everyday life. For example, gravitational force helps to explain why an apple falls to the ground when dropped from someone’s hand. The intimate connection of science with the human experience is what allowed Piaget to use science as a domain for analyzing human cognitive development, because experience interacting with the world leads children toward becoming operational in that world. “The challenge [for children] is to connect experiential learning with the structure of a scientific domain of knowledge through the representational use of verbal symbols and other representational modalities” (Nelson, in press, p. 10). As is implied by Nelson’s statement, the primary goal for the young child, and adults for that matter, is to represent experiences in ways that accurately reflect how the individual perceives them. However, representation is not the default mode of thinking in infancy (Nelson, in press; Piaget, 1936/1977), rather it gradually develops and is gradually learned. From the age
of 4 or 5 years old, children begin to understand that there are different systems of external communication, and they are aware of some of the constraints and affordances that each system provides, such as with drawing versus written language (Tolchinsky-Landsmann & Karmiloff-Smith, 1992). Children possess an understanding that some systems convey certain ideas better than others. We must also consider that different people might find particular systems better matched to them than others. The “affordances” of a system could be thought of on two levels. There might be “idealized universal” affordances; for example, drawing may be a better system for representing a house than written language. However, there may also be an individual level affordance having to do with individual characteristics and preferences (for instance, I would probably not choose music as a medium to express emotion, but someone else might do this very effectively). As children become more aware of symbols and of systems of representation, their task becomes that of combining the use of the symbols with the particular rules of each system to effectively communicate understanding. In other words, children must develop a function for each form of communicating (Munn, 1998). If we study children’s attempts to find function across multiple systems of representation, one can see how this interplay between rules and symbols may develop. Therefore, a review of the literature on how children come to learn representations, and how they learn and use multiple systems of representation, is imperative to the ideas presented here.

5.1 System Rules vs. System Elements

Studies of young children learning the written number system highlight important aspects of children’s attempts to understand the forms of representation they encounter and how to use
Alvarado and Ferreiro (2002) found that 5 and 6-year old children demonstrate an ability to conventionally write single digit numbers. Single digit numbers are constructed from ten basic elements (including zero, although it may pose problems for many children) and do not require a rule, other than conventional means for orienting each element. However, when asked to write two-digit numbers, Alvarado and Ferreiro (2002) found that children invent new solutions for representing these unfamiliar numbers. They found that children of this age rotate the first number of the two-digit sequence (see Figure 8). The students recognized that the first digit in the two-digit number was different than that same element in a single digit number. A three is different than a “thirty,” and thus the children invent a way of using the element “three” to denote the difference between one and two-digit numbers. Two-digit numbers use the same ten elements with the addition of place value as a rule for combining elements. Children who have not yet learned the rules of this system feel compelled to demonstrate the difference between a three and a thirty. Essentially, children make explicit the rules which adults use implicitly. Alvarado and Ferreiro (2002) report finding similar rotations of elements across multiple cultures, suggesting that this trend is not culturally specific (their own research was carried out with Mexican children). Brizuela’s (2004) findings demonstrate a similar

![Figure 8. Recreation of children’s attempts to write one and two-digit numbers, showing the tendency to rotate the first digit in two-digit sequences to denote the difference between three and thirty (Alvarado & Ferreiro, 2002).](image-url)
relationship between element and rule. Children are indeed aware of systems of representation as well as the symbols that comprise those systems, however, their conceptual dilemmas arise as they attempt to represent ideas with which they are uncomfortable. Brizuela (2004) reports on the introduction by a young child of “capital numbers,” a term used in reference to the first digit in a two-digit number. The young child, not knowing that a digit in the tens place of a numeral was called, referred to it as a “capital number.” As with the first letter of a sentence or a proper noun, which is capitalized in the English language, the student “borrowed” this term in attempts to conceptualize two-digit numbers (Brizuela, 2004). While children may tend to have a conventional oral representation of the number (i.e., the child will say “thirty six”, not “three six”), their understanding of the rules of the written system may not be fully developed. As a result, the child invents a solution to the problem: using “capital numbers” to refer to the tens digit in a two-digit number. Brizuela (2004) also eluded to a larger issue, which is that “learning and constructing knowledge involve inventions – novel productions we create, using our present cognitive structures, while trying to make sense of a situation or phenomena” (p. 35). The child’s efforts to borrow from one system to make sense of another ultimately leads to a constructed understanding of the particular rules of a given system. Thus, while children are aware of the various symbols and features of different systems of representation, their invented use of these elements is where they can find meaning.

The ability of children to borrow from one system of representation for application toward another system is further evidence of the importance multiple representations has in science learning. As is demonstrated with the case of capital numbers (Brizuela, 2004), the use of multiple systems of representation is of generative value in constructing knowledge.

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22 Yerushalmy (1997) also calls this “manipulating the conventions” (p. 452), where students find new meaning in conventional representations to suit the needs of a particular problem.
(Bamberger, 1991). Many researchers have attempted to characterize the progress of representational development, and I will review some leading theories before presenting studies of how children learn to represent in multiple modes of expression.

5.2 Mechanisms for Learning to Represent Knowledge

Several researchers have suggested that the process children (and adults) go through in learning to represent knowledge in a new system is cyclical or iterative (e.g., diSessa et al., 1991; Enyedy, 2005; Karmiloff-Smith, 1990; Lehrer & Schauble, 2002). As children continually express knowledge, they invent ways of using symbols and notations until eventually, they find meaning in the conventional ways of representing (diSessa et al.; Enyedy, 2005). Gradually, children are able to re-present knowledge in increasingly abstract ways (Karmiloff-Smith, 1990), and this process of iterative or cyclic expression and re-presentation serves as a model for how students learn to use and appropriate representations. Therefore, progressive symbolization (Enyedy, 2005; Lehrer & Schauble, 2002), described above, is a highly plausible mechanism for representational change. Karmiloff-Smith’s (1990) idea of representational redescription, where children undergo successive revisions of representations en route to developing competency is another possible change mechanism.

In the diSessa et al. (1991) account of inventing graphing, the authors introduce a construct for thinking about how children are able to generate and refine successive representations. diSessa et al. call children's natural ability to represent ideas "Meta-representational Competence" (MRC, expanded in diSessa & Sherin, 2000). They claim that the majority of research in representation focuses on errors students make in attempting to learn
conventional methods, which gives no credence to their natural ability to invent representations. Therefore, MRC (specifically the \textit{meta} aspect) is adopted to validate whatever form a student's representation takes as a building block to more sophisticated representations. MRC encompasses students' ability to not only \textit{invent} representations (diSessa et al.; Sherin, 2000) but to also \textit{critique} representations (Azevedo, 2000). As many studies have shown (Brizuela, 1997; 2004; diSessa et al.; Enyedy, 2005; Lehrer & Schauble, 2000b, 2002; 2004; Penner, Giles, Lehrer, & Schauble; 1997), children undoubtedly have an ability to invent representations, completely irrespective of the rules of specific symbol systems (Brizuela, 2004; Lehrer & Schauble, 2000a). Children’s natural ability to critique a representation is similar to the concept of children re-constructing links between a conceptual idea and an externalization. Additionally, engaging in this act of refinement while in a social setting, where one is motivated by the referential-communicative aspects of representation (Tolchinsky-Landsmann & Karmiloff-Smith, 1992), could be a source of conceptual development. Nemirovsky (1994) reports on a case study of Laura, a high school student trying to make sense of the “velocity sign.” When motion is represented as a quantity, the sign (either negative or positive) indicates directionality. For many students, this concept is confusing and non-intuitive (Nemirovsky, 1994); however, for Laura, the act of making sense of this representation served as an illustrative episode of the role of critique in coming to understanding conventional notations.

Laura’s understanding of concepts of distance, speed, direction, and position and their relationships were all sufficient (Nemirovsky, 1994). Her trouble arose in trying to make sense of the velocity sign, which was a representation introduced by the particular motion graphing software and hardware used in the study. Ultimately, the story of Laura’s learning is an example of “becoming fluent with a type of symbol-use” (Nemirovsky, 1994, p. 418). A crucial aspect of
learning to operate within a particular system of representation is learning to use that system’s rules. However, Nemirovsky (1994) uses the case of Laura to demonstrate that rule-use is but one aspect of a broader perspective that includes re-describing what one already knows in order to make sense of the new, unfamiliar representation system. The importance of students’ inventing ways of representing concepts was shown with the case of capital numbers to denote the first digit in a two-digit number (Brizuela, 2004). With that example, children invent a use for a known rule such that they can explore a new, unfamiliar situation. Having recognized the need for distinguishing between a single digit and the first element in a two-digit sequence, children borrow a rule from another system to help with the differentiation. In doing so, this unconventional use of a rule allows students to explore a new situation, that eventually leads them to understanding the conventional ways of representing two-digit numbers. Thus, inventing ways of representing helps the child understand the system and the concept. Perhaps of equal importance is one’s ability to critique representations as means for refining one’s understanding. A critique of an invented form, such as with inventing ways of graphing (diSessa et al., 1991) or mapping (Enyedy, 2005), leads students to find ways of expressing their understanding in a new system (Nemirovsky, 1994). Children’s attempts to learn and understand ways of symbolizing is multi-faceted and involves aspects of invention, critique, and eventually adopting convention. Such learning activities can involve existing, conventional symbols or the idiosyncratic notations that children produce in attempts to share knowledge. Either way, this milieu of conventions, inventions, and finding meaning makes the process of how children represent knowledge and how they appropriate these representations of critical importance to education research. Given that we have established science as a domain of multiple
representations, work done on how students create and find meaning across many systems can provide a powerful lens for investigating scientific knowledge through the eyes of representation.

5.3 Multiple Representations

The importance of invented representations and critiquing externalizations in the process of developing understanding of mathematics and science concepts has been articulated; however, explicating across multiple systems deserves further attention. According to Pérez-Echeverría and Scheuer (in press), “The use of alternative external representations to describe a single situation assists the explication of epistemic attitude, across developmental periods, learning situations and domains of knowledge” (p. 11). Others have made a similar argument, where attempts to express knowledge across systems of representation are shown to be beneficial for conceptual understanding (Brizuela & Earnest, 2007; Lehrer & Schauble, 2002). One specific reason for the benefit of multiple representations lies in the possibility that each system of representation may highlight aspects of a problem that others do not (Kaput, 1998; Pérez-Echeverría & Scheuer, in press; Zhang & Norman, 1994). That particular conceptual features are made more salient in certain systems, and that specific types of reasoning are supported by specific types of representations are both stances that are supported by the literature in both mathematics and science education.

A distinction is necessary prior to the review of the literature on this topic. External representations are used in this section in two different manners: expressively and interpretively (Toth, 2000). An expressive representation is created by an individual in attempts to convey information to oneself or to a broader audience. These externalizations can be idiosyncratic,
conventional, or a combination of both. Interpretive representations are given to the individual, such as with many computer software applications or in the case of Laura and the velocity sign (Nemirovsky, 1994). Regardless of the particular type of externalization, the general tenets of how children learn to use and appropriate these representations hold relatively consistent. Therefore, studies involving both interpretive and expressive representations will be included in this review.

The research in science and mathematics education has demonstrated that students can develop richer conceptual understanding and deeper knowledge of representational practices when the educational activities involve the use of multiple representations. Specifically, the variety of perspectives inherent in the many forms of representation provide for variance in which aspects of the problem are made apparent, which fosters deeper insight for children (Confrey, 1991). Highlighting conceptual elements of the same problem can guide students to uncover patterns across representations. Complex language and circumstantial issues can also be unpacked by clarifying aspects of the problem through different modes of expression (Schwartz & Yerushalmy, 1995). The use of multiple representations to increase insight has been shown to specifically help students grasp concepts like mathematical similarity (Lehrer, Strom, & Confrey, 2002), graphical notations of algebraic concepts (Schwartz & Yerushalmy, 1995), how to model plant growth (Lehrer & Schauble, 2002), and position versus time graphs (Nemirovsky, Tierney, & Wright, 1998; Thornton, 1987). As Bamberger (1991) suggests, for many individuals (students and teachers), unpacking the problem involves the exploration of many forms of that problem; that is, “different kinds of conceptualizations…can be explored by navigating across different representations of the same problem” (Brizuela & Earnest, 2007, p. 299).
Zhang and Norman (1994) introduced the concept of the *representational effect* to address this issue of different representations linked to different conceptual aspects. They define the effect by saying that “different isomorphic representations of a common formal structure can cause dramatically different cognitive behaviors” (Zhang & Norman, 1994, p. 88; see also Kaput, 1998). In studies investigating well-known problems like tic-tac-toe (Zhang, 1997) and the Towers of Hanoi (Zhang & Norman, 1994), they were able to show that individuals showed varying ability to solve the problem based on the representation presented. Parnafes and diSessa (2004) found similar results when investigating motion through two different representations with middle school students. Their findings suggested a difference in reasoning styles based on the type of representation on which the students focused. In these examples, the students were charged with interpreting representations that were presented to them in computer software environments (Parnafes & diSessa, 2004; Zhang, 1997; Zhang & Norman, 1994). Interpreting and making-sense of representations across multiple systems has been shown to have an impact on students’ understanding of math and science ideas. However, gains in understanding have also been shown when students generate the representations across multiple systems. While some have shown the power of *interpretive* uses of representations, *expressive* uses of representations are also powerful contexts for impacting student understanding (Toth, 2000).23

Brizuela and Earnest (2007) report on elementary school students learning algebraic concepts. The students were presented with a problem and asked to first verbally articulate their view of the problem, then put on paper some representation of the problem, then generate tables, and finally discuss graphical representations. The researchers found that as the students progressively represented the problem, “the explicit and implicit qualities of notations

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23 It is important to note that educational activities do not have to be exclusively interpretive or expressive. In fact, combinations of the two forms may be the most effective way for children to explore new conceptual areas.
continually refined and enhanced their understandings of the problem” (Brizuela & Earnest, 2007, p. 282). Brenneman, Massey, Machado, and Gelman (1996) showed similar results in algebra, whereby when students were introduced to new representations for problems, their frequency of use for each type increased when searching for solutions. Students appeared to show an increase in representational flexibility (Karmiloff-Smith, 1992) as they found meaning in each of the systems. Tytler and Prain’s (2007) study in the domain of science confirmed these previous findings, showing that as students constructed multiple representations of evaporation, their conceptual understanding increased. They attribute the gains to “a shift in [the students’] capacity to imagine the process whereby water can exist in air, involving the construction of a narrative of causation allied with the spatial representation” (Tytler & Prain, 2007, p. 244). The common theme running through these studies is that the exploration of concepts by students generating multiple representations results in increased conceptual understanding. However, the results of this work are contingent on a crucial principle for all educational activities, which is that educationally rich activities require posing appropriate questions such that the investigation of the concept in each of the systems yields a benefit (Friedlander & Tabach, 2001). Children can pose appropriate questions as well as teachers, with the appropriate guidance and structuring (Lehrer & Romberg, 1996). No matter who generates the problems, their appropriateness in addressing the concepts at hand must be considered. Just as the representational effect (Zhang, 1994) suggests, not all representations are best suited for all problems and thus, before children have gained representational competency, care should be taken in selecting the kinds of problems used to help teach children how to externalize knowledge.
6.0 Conclusions

The driving force behind this work is to define the importance of and argue for the centrality of externalizing ideas in the learning of science. Science is a domain inundated with different systems of representation and, thus, children’s attempts to understand scientific ideas are dependent on their ability to learn and appropriate representations. As Piaget and others have suggested, external representations of knowledge are essential aspects of thought. Lucienne’s kinesthetic gesture highlights how representing and action can help a child eventually perform said action. Children acquire the ability to represent ideas, actions, and experiences at a young age (Piaget, 1936/1977), and the capacity for representation arms children (and adults for that matter) with tools for investigating new ideas and experiences in new ways. The case of Richard Feynman’s introduction of a new representation to the field of theoretical physics highlights the importance of representations as tools for developing thought.

For children, learning the written number system or graphical notation is a process whereby interactions with new system elements and with rules for combining elements complement the construction of knowledge around the particular content of interest. The process of representing not only supports children in understanding the system of representation, but it also supports their understanding of the particular concepts under consideration. Once the child has come to understand the elements and rules of a system of representation, the artifacts produced when externalizing ideas in that system become conceptual objects in their own right. Just as with the Feynman example, once his colleagues came to understand exactly how he constructed these diagrams, the figures became representations of the concepts, not simply a process for solving a problem. As children develop, they begin to shift from a process perspective of representation to one where external representations become objects on which
they operate. This multi-faceted experience of learning to represent knowledge plays a crucial role in the construction of science understandings.

Through mechanisms such as progressive symbolization (Enyedy, 2005) or representational redescription (Karmiloff-Smith, 1990, 1992), students iteratively link their ideas to what they produce in the external world. With idiosyncratic and spontaneous representations, students are building an artifact from the idea they harbor in their mind. However, attempts to become operational with conventional systems of representation (e.g., the written number system, graphing, or written language) require the child to link the conventional elements and rules to his or her understanding of the underlying concept. As with Laura and the velocity sign (Nemirovsky, 1994), aligning one’s understanding to ways of representing that understanding takes time and likely results in different “linkages” for each individual. To compound the matter, conventional systems of representation are developed within some cultural milieu, and the role of culture in the learning and appropriating of these systems is one that cannot be overlooked. The complex integration of theoretical constructs in the preceding section addresses the first guiding question set forth at the beginning of this work, “Why study representation?” The importance and centrality of external representations to thinking and learning, particularly in the domain of science, make representation and important aspect of constructing knowledge. For this reason, it is imperative that researchers continue to investigate the role of representation in developing science understanding.

The second guiding question aims to define what, exactly, is meant by representation. The term representation is used broadly in the social sciences with a number of connotations and implications. In this work, it means an external artifact (either a material artifact or some measure of utterance or gesture) that establishes a “stand for” relationship. In other words, the
external representation stands for the idea that an individual wishes to express (either to him- or herself, or to another individual). For research and discussion purposes, the myriad possible externalizations are classified as systems of representation, which require the existence of elements and rules for combining elements. Obviously there exists varying degrees of rule and element structure, but as a construct, this notion of rules and elements serves to help researchers understand how children may come to understand conventional ways of externalizing knowledge. Another application of the notion of systems of representation is in the historical accounts of how conventional methods of representation evolved over the course of human history. More specifically, how these means of expression impacted the nature of knowledge and the concepts being represented.

A recount of the evolution of representations over time is relevant to how children learn conventional representations because it provides a perspective through which to analyze their attempts to make sense of and appropriate these ways of externalizing. It is not to say that children re-invent the conventional systems that have evolved, but rather history reveals the major challenges and advancements of representation. These same conceptual hurdles may be the very areas that challenge children as well. For example, as Olson (1994) lays out, the development of written notation helped to provide syntax for speech patterns. Therefore, it is plausible that children’s attempts to write ideas on paper may help them clarify and process the ideas they hold. Olson’s example of the development of a way for writing “three sheep” that avoided the repetition of three identical characters exemplifies this concept of representation facilitating information processing. And in fact, children follow a similar developmental path when learning about the written number and place value systems. Therefore, the history of how
conventional systems of representation developed and how that evolution changed the nature of knowledge can enlighten attempts to understand how children represent their ideas.

The final guiding question of this work was to review empirical data on how children learn and appropriate representations. A great deal of literature, particularly in mathematics education, has addressed the role of representation in learning. An interesting line of research attempts to regard modes of expression as systems of representations. This work has shown that students must learn both the elements of the system (e.g., written number notation, coordinate axis for graphing) and the rules for combining these elements (e.g., place value system). Students tend to “borrow” rules and elements from systems in attempts to represent their understandings before they are knowledgeable of how the system elements and rules relate. This suggests that students have rich conceptual ideas prior to the mastery of conventional means for expressing them. The literature also highlights potential mechanisms for learning and eventually appropriating external representations. Notions such as progressive symbolization, representational redescriptions, and meta-representational competence (diSessa & Sherin, 2000) help researchers make sense of the progression students experience in learning conventional systems. An important aspect of all this work is the role of invention in coming to understand concepts and ways of representing them. Encouraging children to invent representations of what they know provides them with a foundation for linking their knowledge to conventional methods. Students must link their knowledge to new elements of expression; and, furthermore, working from their natural (i.e., idiosyncratic and spontaneous) tendencies guides children toward constructing an understanding of more normative concepts and representations. Just as scientists use multiple forms of representation in expressing, discussing, and critiquing developments in their fields, the use of multiple systems of representation by children in constructing knowledge
is also highly beneficial. However, the literature on the use of multiple, student-generated representation appears to be better developed in mathematics education than in science. Thus, more research is needed into how children spontaneously represent their knowledge across multiple systems.

7.0 IMPLICATIONS AND FUTURE WORK

Both the theoretical positions and the empirical data presented here highlight the importance of representations in science. Furthermore, the literature from mathematics and science education suggests that a great deal of power lies in providing students with opportunities to spontaneously represent knowledge. Throughout history, representation has played a prominent role in the development of scientific concepts. Just as the domain has progressed over time, children, too, undergo a developmental progression with regard to the ability to externalize knowledge. This progress involves the invention, critique, and re-appropriation of myriad forms of expression. While we hope that students eventually come to know and use conventional methods of representing information in science, we must acknowledge that in order for students to find meaning in these systems, they must first be given opportunities to construct idiosyncratic means of expression. There exists a great deal of compelling work showing the power of providing students with opportunities to invent spontaneous ways of representing their understanding (Brizuela, 2004; diSessa et al., 1991; Enyedy, 2005; Lehrer & Schauble 2002), and it is my belief that this work needs to be expanded.

The literature on children inventing representations suggests that through this process, students come to understand the need for conventional rules and conventional elements.
Alongside learning and appropriating the conventional systems of representation comes the development of scientific understanding as well. In the science education literature, evidence of how specific systems of representation support the learning of particular aspects of science is underdeveloped. The literature illustrates the power of using multiple systems of representation in helping students understand concepts in science. However, much of this work involves situations where students are asked to make sense of representations that are given to them. As Zhang and Norman (1994) suggest with their construct of a representational effect, the system of representation may impact the conceptual understanding of the child. Similarly, students likely represent their ideas differently in different systems. Therefore, I believe researchers must work to identify the ways in which students explore scientific concepts in different systems of representation, conventional and idiosyncratic. Work is needed that capitalizes on the power of inventing representations (as opposed to interpreting representations generated by someone else) for students in fundamental areas of science. Investigations into how students spontaneously represent ideas across multiple systems can potentially provide significant insights. With this new understanding, we may begin to construct meaningful learning environments that allow students to learn conventional forms of representation, while at the same time develop conceptual understanding.

More specifically, I believe research is needed to investigate how students represent static and dynamic processes in different systems. For example, a drawing is a static entity, however, it is often used as the preferred system through which students represent dynamic processes. Perhaps there are alternative systems of representation that may aid the development of understanding of dynamic processes by allowing students to better externalize process-oriented conceptions. Additionally, I believe the process of representing helps to refine and organize
thought. Therefore, further research into how students explain ideas in science through multiple forms of representation, attending specifically to how these explanations evolve throughout the process, would be beneficial to the field. Ultimately, the domain of science is dependent on the development of models which consist of external representations. Therefore, fundamental research into how students spontaneously represent their understanding of scientific concepts while finding meaning in the conventional means of expressing ideas is of vital importance to the field of science education.
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